Math Links

PROFICIENCY CHALLENGES
GRADE 7
SETS 5-8

Proficiency Challenges are sets of interesting, mixed-topic problems. It may take a lot of time to complete each set, so consider doing only one or two parts at a time.

Name_____ Period _____ Date _____

PROFICIENCY CHALLENGE 5

1. If both a and b are positive integers, determine if the following expressions have positive or negative values. Explain your reasoning.

   $(a)(b)$ $(-a)(-a)(-b)(-b)$ $\frac{(-a)(b)}{(-b)(a)}$ $\frac{(-a)(-b)}{(-b)(-a)}$

   If both a and b are negative integers, determine if the expressions above have positive or negative values. Explain your reasoning.

2. Consider the given expression and the given values of k. For which of these values of k does the given expression represent a rational number?

   Given expression: $\frac{k+5}{-2+k}$ Values for k: 5, -5, 2, -2

3. Create numbers that meet the given criteria. Write your answers as rational numbers in the form $\frac{a}{b}$ where both a and b are numbers from the list below. You may use a number as many times as you would like.

   -3 2 500 -7 8 9 16

   a. Write three rational numbers that each have a value less than -1.

   b. Write three rational numbers that each have a value greater than 0 but less than 1.

   c. Write two rational numbers that each have a value between -1 and -2.

   d. Write a rational numbers that has a value between 3 and 10.

   e. Write the least possible rational number.

4. Consider the equation $y = \frac{1}{x}$. Substitute several values **greater than 1** for x, and find y. Organize your work in a table. Then answer the following questions.

   (For example, if $x = 2$, then $y = \frac{1}{2}$)

   As x becomes greater and greater, what appears to happen to the value of y?

   Jason thinks that if x increases enough, y will eventually equal 0. Do you agree or disagree with Jason's conjecture? Explain your reasoning.

   Christina says, "As the value of x becomes really big, the value of y gets closer and closer to 0, but y will never equal zero." Do you agree or disagree with Christina's conjecture? Explain your reasoning.

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PROFICIENCY CHALLENGE 6

1. Simplify the following expressions.

   a. $\dfrac{\left(\frac{1}{2}\left(6 + (3^2)(6)\right)\right)}{(2)(5^2) - 2(2^3 + 2)}$

   b. $2^{\left(\frac{(5-2)^3}{3^2}\right)}$

   c. $\dfrac{1\frac{3}{4}}{\frac{6}{5}} \div \dfrac{\frac{7}{3}}{3\frac{1}{2}}$

2. William was quizzing Wanda on her math skills. He told her to add 2 and 6, then triple the sum, then divide her answer by 12.

   She wrote down the following expression: $(2 + 6)^3 \div 12$

   What was Wanda's mistake? Write a numerical expression that matches William's statement to Wanda. Then write a verbal description that matches Wanda's numerical expression.

3. Use numbers and math symbols to write the following as a single expression:

   Subtract 3 from 7. Then square that number and add it to the difference of 10 and the cube of 2.

   Share your expression with a partner. Are they equivalent? Explain. Then calculate each other's expressions.

4. Lance and Jerome are driving through the desert of Nevada and they are running low on gas. They have been averaging 27.4 miles per gallon. They have 132.7 miles to the next gas station and have 4.8 gallons of gas in left. Will they make it to the gas station? Explain.

5. Mo, Larry, and Curly are working to paint Mr. Marshall's house. Mo works 6.5 hours, Larry works $3\frac{2}{3}$ hours, and Curly works $1\frac{5}{6}$ hours more than Mo and Larry combined. Mr. Marshall paid them $330 total. How should they divide the money fairly between them based on the hours they worked? Round your answers to the nearest dollar.

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PROFICIENCY CHALLENGE 6 (Continued)

6. Determine if the following expressions are positive or negative. Explain your reasoning.

   a. $(-1)^5$ b. $(-1)^6$ c. $(-1)^{100}$ d. $(-1)^{99999}$

   Explain how you know whether $(-1)^n$ is positive or negative for different whole number values of n.

7. Each of the following sets of ordered pairs represents three vertices of a rectangle. Find the missing coordinates of the fourth vertex. Try to use numerical reasoning instead of graphing.

   Rectangle *ABCD*

   A (1, 1.2) *B* (1, -0.4) *C* (___, ___) *D* (2, 1.2)

   Rectangle *EFGH*

   E $\left(-1\frac{1}{5}, -\frac{4}{5}\right)$ *F* $\left(\text{—}, \text{—}\right)$ *G* $\left(\frac{3}{5}, -1\frac{3}{5}\right)$ *H* $\left(\frac{3}{5}, -\frac{4}{5}\right)$

   Which of the two rectangles has the greater area? Justify your conclusion.

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PROFICIENCY CHALLENGE 7

1. Read each of the following statements. For each statement, determine if it's correct or not and then fully explain your reasoning.

   - Statement 1: On Friday, it will rain or it will not rain. Since there are two outcomes, there is a probability of 0.5 that it will rain.
   - Statement 2: Because Mr. and Mrs. Smith have three boys, their fourth child is more likely to be a girl.
   - Statement 3: Rolling two dice and getting a sum of 2 is half as likely as rolling two dice and getting a sum of 3.
   - Statement 4: There are three possible outcomes for a soccer team playing in a match. They can win, lose, or tie. The probability of winning therefore is $\frac{1}{3}$.

2. Janice wants to hold a fundraiser. She sells raffle tickets like the one below for $1 to fans at a basketball game.

1	3	A	C
2	4	B	D

   At the end of the basketball game, she asks every person to circle one number and one letter. For example, a person could choose combinations like 1A, 3D, or 4D.

   After everyone has circled his or her numbers, Janice randomly draws a number (1-4) from one hat and draws a letter (A-D) from another hat.

   Anyone who circled the same number-letter combination that Janice draws wins $10.

   a. Is it likely that Janice will make money on this fundraiser? Explain your reasoning.

   b. Yolanda thinks that Janice should offer $20 to winners so more people will buy tickets. How would this affect Janice's likelihood of making money?

3. Janice's friend, Alice, wants to design a different card for which the chance of winning is more difficult. Draw the card and explain why Alice's card has a lesser chance of winning.

4. Hank said, "If I flip a plastic cup 100 times, I predict it lands open side up about a third of the time, open side down about a third of the time, and on its side about a third of the time."

   a. Before trying this experiment, respond to Hank's prediction.

   b. Try this experiment, and discuss whether your results agree or not with Hank's prediction or your initial thoughts.

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PROFICIENCY CHALLENGE 8

1. Riley said, "If I multiply $\frac{x}{6} + \frac{2}{3}$ by 6, I get $x + 4$. Therefore, $\frac{x}{6} + \frac{2}{3}$ and $x + 4$ are equivalent expressions." Use reasoning to explain or examples to show that Riley is incorrect.

2. Show that none of these expressions are equivalent.

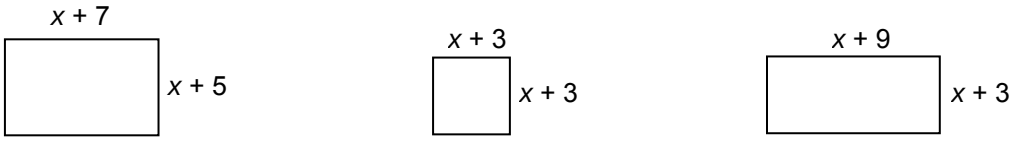
   $2[4 + 2 \bullet 3 + 4]$ $2[4 + 2(3 + 4)]$ $2[(4 + 2)(3 + 4)]$

   Can a set of grouping symbols in any of the expressions above be removed without changing the value of the expression? Explain.

3. Determine which of the following expressions are equivalent to $2(3x + 2)$.

$6x + 2$	$2(2 + 3x)$	$3x + 2 + 3x + 2$	$2(x + x + x + 1 + 1)$	$x(2) + 2(2)$	$(3x + 2)(3x + 2)$

4. Find the perimeter of the rectangles below. The rectangles are not drawn to scale.



   Do any of the rectangles have equivalent perimeters for any given value of x? Explain.

5. William finds the perimeter of a rectangle by adding the length and the width and then doubling this sum. Matthew finds the perimeter of a rectangle by doubling the length, doubling the width and then adding the doubled amounts.

   Write an expression that shows how William finds the perimeter.

   Write an expression that shows how Matthew finds the perimeter.

   Explain why their expressions are equivalent.

6. Valerie has a room with a wall that is $10\frac{3}{4}$ feet wide.
   - She wants to hang a TV in the middle of the wall that is 3.25 feet wide.
   - She wants to hang one picture on each side of her TV that are both the same size.
   - She wants 1-and-one-quarter feet between the TV and each picture.
   - She wants to leave 6 inches between the outer wall edges and each picture.

   Find the maximum width of **each** picture.